A Survey of Payload Integration Methods

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Nomenclature

$\{F_e^P\}$ = payload member force vector
$\{F_p\}$ = payload force vector
H, L = higher and lower frequency, respectively
$[I_B],[I_P]$ = $\begin{bmatrix} I \\ 0 \end{bmatrix}$, defined by Eqs. (10) and (11)
$\{k_e\}$ = stiffness matrix of element e
$[K_P]$ = payload stiffness matrix
$[L_B]$ = $\begin{bmatrix} 0 \\ I \end{bmatrix}$, defined by Eqs. (10) and (11)
$[M_P]$ = payload mass matrix
$\{q_B\},\{q_I^B\},\{\bar{q}_N\}$ = generalized coordinate vectors defined by Eqs. (20), (17), and (16), respectively
$\{R_I\}$ = interface reaction force vector
$[S_B]$ = booster matrix defined analogously to $[S_P]$
$[S_P]$ = noninterface portion of payload constraint mode matrix; defined by Eq. (9)
$[T_B]$ = $\begin{bmatrix} S_B \\ I \end{bmatrix}$, matrix of booster constraint modes
$[T_e]$ = payload displacement to element displacement transformation matrix for element e
$[T_P]$ = matrix of payload constraint modes
$\{x\}$ = vector of displacement coordinates
$[\alpha]$ = perturbation matrix expansion coefficients defined by Eq. (40)
ϵ = perturbation parameter employed in Eqs. (35) and (36)
$[\phi_B]$ = matrix of free-interface booster normal modes
$[\phi_B^R]$ = matrix of residual attachment modes defined by Eq. (28)
$[\phi_I^B]$ = matrix of interface normal modes based on the interface portions of Eq. (15); also, interface portion of $[\phi_B]$
$[\bar{\phi}_N]$ = matrix of fixed-interface normal modes

Superscripts and Subscripts

B, P = booster and payload, respectively
F, R = response due to external forces and payload feedback, respectively
I, N = interface and noninterface, respectively

I. Introduction

As the Space Shuttle takes its place as the major U.S. space transportation system, there is renewed interest in the development of improved methods for predicting Shuttle payload dynamic loads.[1] Present analytical techniques by which design loads are predicted are very costly and time-consuming (engineering time and turnaround time) since a typical load cycle generally requires generation of a payload model and booster model, calculation of the modal characteristics of the payload and booster, coupling of the payload to the booster and calculation of system modal characteristics, calculation of the time history of the response of the system to specified forces, and use of the response results to calculate member loads. Although methods for carrying out such full-scale load cycles are well developed, methods requiring less time and computational effort are not nearly so well developed and tested.

The objective of the present paper is to survey some of the most prominent "full-scale" and "short-cut" methods of payload load prediction. A full-scale method is defined as a method which requires a solution of coupled booster/payload equations without making approximations other than modal truncation. Short-cut methods, on the other hand, involve further simplifying assumptions which make it possible 1) to avoid direct solution of the coupled equations of the booster/payload system, and 2) to avoid, as much as possible, the interfacing between different organizations. Thus, short-cut methods are primarily intended to make it possible to evaluate the effect of small changes in the payload in a relatively short time.

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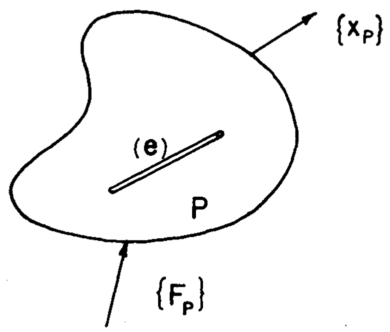

Fig. 1 Payload with element e identified.

Most payload load analysis methods rely on some form of component mode synthesis. As a result, the literature on component mode synthesis, as reflected in previous survey articles,[2-4] is relevant. In addition, numerical comparisons of various component mode synthesis methods have appeared.[2,3,5-8] The full-scale methods in the present paper deal only with those methods which these previous studies showed to be the most promising. On the other hand, there have been several recent developments related to substructuring, or component mode synthesis, such as new techniques for computing system modes of undamped[9,10] and damped[11,12] systems. These topics are beyond the scope of the present paper.

In the following sections the system equations of motion for a coupled booster/payload system will be presented and their solution by component mode synthesis, direct integration, and transformation to the frequency domain will be indicated. A number of short-cut methods will be presented in order to give some indication of the current state-of-the-art with regard to development of such methods. Finally, directions for further research and development will be suggested. This paper is a condensation of an extensive survey report on payload methodology.[13]

II. Member Loads

Figure 1 represents a payload with a typical member "e" whose dynamic "load," or internal force vector, is to be determined. This can be expressed as

$$\{F_e^P\} = [k_e][T_e]\{x_P\} \tag{1}$$

where $\{x_P\}$ is the displacement of the payload, $[k_e]$ the stiffness matrix of element e, and $[T_e]$ the transformation matrix which gives the displacements of element e in terms of the payload displacements, which are essentially determined by the equation*

$$[M_P]\{\ddot{x}_P\} + [K_P]\{x_P\} = \{F_P\}_{\text{total}} \tag{2}$$

It has been suggested[14] that the mode-acceleration approach leads to more accurate values of $\{x_P\}$, and hence of the internal loads $\{F_e^P\}$. The essence of the mode-acceleration approach is to solve Eq. (2) for $\{x_P\}$ and perform modal expansion of $\{\ddot{x}_P\}$.[15] That is,

$$\{x_P\} = [K_P]^{-1}[\{F_P\}_{\text{total}} - [M_P]\{\ddot{x}_P\}] \tag{3}$$

Hruda and Jones[14] have developed a more complete derivation of the mode-acceleration form for component (payload) internal loads and have indicated which component mode synthesis methods lend themselves to this approach.

*Throughout most of this paper, damping is omitted from equations of motion. It is assumed that standard modal damping techniques could be employed to include damping.

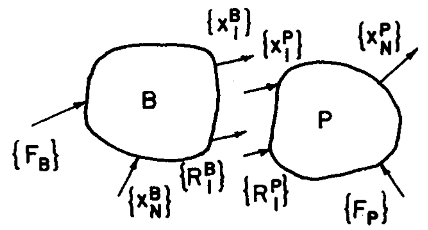

Fig. 2 Freebody diagrams of booster B and payload P.

III. System Equations of Motion in the Discrete Time Domain

A. Uncoupled Equations of Motion

The objective of this section is the derivation of the equations of motion of the booster/payload system. Figure 2 shows symbolic freebody diagrams of the booster B and the payload P. The booster and payload are connected to each other through the interface. Physically, the interface is the collection of structural "hard" points which the booster and payload have in common. Mathematically, this means that the generalized displacement vector $\{x_I^B\}$ on the booster side of the interface must be equal to its equivalent $\{x_I^P\}$ on the payload side. Hence,

$$\{x_I^B\} = \{x_I^P\} \tag{4}$$

for *all* times t. Similarly, the generalized reaction vectors $\{R_I^B\}$ and $\{R_I^P\}$ at the interface satisfy

$$\{R_I^B\} = -\{R_I^P\} \tag{5}$$

for *all* times t.

From the freebody diagrams in Fig. 2 we can easily write the equations of motion for the booster B and the payload P as

$$\begin{bmatrix} M_B & \\ & M_P \end{bmatrix} \begin{Bmatrix} \ddot{x}_B \\ \ddot{x}_P \end{Bmatrix} + \begin{bmatrix} K_B & \\ & K_P \end{bmatrix} \begin{Bmatrix} x_B \\ x_P \end{Bmatrix} = \begin{Bmatrix} F_B \\ F_P \end{Bmatrix} + \begin{Bmatrix} 0 \\ R_I^B \\ 0 \\ R_I^P \end{Bmatrix} \tag{6}$$

where $\{x_B\}$ represents the generalized displacement vector of B, $[M_B]$ the mass matrix, $[K_B]$ the stiffness matrix, and $\{F_B\}$ the externally applied force. Similar quantities were introduced for payload P. Equation (6) represents the *uncoupled* equations of motion of the undamped booster B and the undamped payload P.

B. Free Booster-Constrained Payload Equations of Motion

In order to derive the equations of motion for the coupled system (i.e., booster/payload system) the a priori unknown reactions $\{R_I^B\}$ and $\{R_I^P\}$ must be eliminated. There are essentially two frequently used transformations that accomplish this objective. The first one makes use of an interface-restrained payload and a *free booster*, while the other transformation adopts a restrained payload, a restrained booster, and a *free interface*. Both approaches make use of the following partition of vector $\{x_P\}$,

$$\{x_P\} = \begin{Bmatrix} x_N^P \\ x_I^P \end{Bmatrix} \tag{7}$$

where $\{x_N^P\}$ represents the noninterface displacements of the payload P. $\{x_N^P\}$ can be written as the sum of two parts,

$$\{x_N^P\} = [S_P]\{x_I^P\} + \{\bar{x}_N^P\} \tag{8}$$

where

$$[S_P] = -[K_{NN}^P]^{-1}[K_{NI}^P] \tag{9}$$

and where $\{\bar{x}_N^P\}$ is the vector of noninterface displacements of P with respect to the interface.

The free booster version of the equations of motion is obtained by using Eqs. (4) and (8) to form the coordinate transformation

$$\begin{Bmatrix} x_B \\ x_P \end{Bmatrix} = \begin{Bmatrix} x_N^B \\ x_I^B \\ \hline x_N^P \\ x_I^P \end{Bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ \hline 0 & S_P & I \\ 0 & I & 0 \end{bmatrix} \begin{Bmatrix} x_N^B \\ \hline x_I^B \\ \hline \bar{x}_N^P \end{Bmatrix} \tag{10}$$

Using the notation implied in

$$\begin{Bmatrix} x_B \\ x_P \end{Bmatrix} = \begin{bmatrix} I_B & L_B & 0 \\ 0 & T_P & I_P \end{bmatrix} \begin{Bmatrix} x_N^B \\ x_I^B \\ \bar{x}_N^P \end{Bmatrix} \tag{11}$$

and employing Eqs. (5), (6), and (10), the following free booster form of the equation of motion is obtained.

$$\begin{bmatrix} I_B^T M_B I_B & I_B^T M_B L_B & 0 \\ L_B^T M_B I_B & L_B^T M_B L_B + T_P^T M_P T_P & T_P^T M_P I_P \\ 0 & I_P^T M_P T_P & I_P^T M_P I_P \end{bmatrix} \begin{Bmatrix} \ddot{x}_N^B \\ \ddot{x}_I^B \\ \ddot{\bar{x}}_N^P \end{Bmatrix} + \begin{bmatrix} I_B^T K_B I_B & I_B^T K_B L_B & 0 \\ L_B^T K_B I_B & L_B^T K_B L_B + T_P^T K_P T_P & 0 \\ 0 & 0 & I_P^T K_P I_P \end{bmatrix} \begin{Bmatrix} x_N^B \\ x_I^B \\ \bar{x}_N^P \end{Bmatrix} = \begin{Bmatrix} I_B^T F_B \\ L_B^T F_B \\ 0 \end{Bmatrix} \tag{12}$$

C. Constrained Booster-Constrained Payload Equations of Motion

If the booster is represented in the same manner as the payload, i.e.,

$$\{x_N^B\} = [S_B]\{x_I^B\} + \{\bar{x}_N^B\} \tag{13}$$

then the appropriate coordinate transformation is

$$\begin{Bmatrix} x^B \\ x^P \end{Bmatrix} = \begin{Bmatrix} x_N^B \\ x_I^B \\ \hline x_N^P \\ x_I^P \end{Bmatrix} = \begin{bmatrix} I & S_B & 0 \\ 0 & I & 0 \\ \hline 0 & S_P & I \\ 0 & I & 0 \end{bmatrix} \begin{Bmatrix} \bar{x}_N^B \\ \hline x_I^B \\ \hline \bar{x}_N^P \end{Bmatrix} \tag{14}$$

The resulting equations of motion for restrained booster, restrained payload, and free interface can be written in the form

$$\begin{bmatrix} I_B^T M_B I_B & I_B^T M_B T_B & 0 \\ T_B^T M_B I_B & T_B^T M_B T_B + T_P^T M_P T_P & T_P^T M_P I_P \\ 0 & I_P^T M_P T_P & I_P^T M_P I_P \end{bmatrix} \begin{Bmatrix} \ddot{\bar{x}}_N^B \\ \ddot{x}_I^B \\ \ddot{\bar{x}}_N^P \end{Bmatrix} + \begin{bmatrix} I_B^T K_B I_B & 0 & 0 \\ 0 & T_B^T K_B T_B + T_P^T K_P T_P & 0 \\ 0 & 0 & I_P^T K_P I_P \end{bmatrix} \begin{Bmatrix} \bar{x}_N^B \\ x_I^B \\ \bar{x}_N^P \end{Bmatrix} = \begin{Bmatrix} I_B^T F_B \\ T_B^T F_B \\ 0 \end{Bmatrix} \tag{15}$$

$[S_B]$ and $[T_B]$ are defined in the same manner as $[S_P]$ and $[T_P]$. The principal difference in Eqs. (12) and (15) is the additional uncoupling of stiffness terms in Eq. (15).

The columns of $[T_P]$ and $[T_B]$ are referred to as "constraint modes." Hurty[16] introduced constraint modes and represented the motion of components by a combination of constraint modes, rigid-body modes, and fixed-interface modes. Craig and Bampton[17] simplified Hurty's method by showing that rigid-body modes did not have to be treated separately from constraint modes, in effect, providing the coordinate transformation of Eq. (8). It is this coordinate transformation which leads to the uncoupling of stiffness terms in Eqs. (12) and (15).

IV. Solution of the Equations of Motion by Component Mode Synthesis

The most straightforward approach to determine $\{x_P\}$ from Eq. (12) or (15) is to employ a step-by-step numerical integration routine. One drawback of this direct approach is the high computational cost due to the large number of degrees of freedom used to describe today's aerospace models. Therefore, some form of component mode synthesis is usually employed to reduce the number of degrees of freedom prior to numerical integration.

A. Component Mode Synthesis Solution

The component mode synthesis method proposed by Craig and Bampton[17] consists in transforming booster and payload noninterface coordinates to modal coordinates through the use of fixed-interface normal modes, i.e., by transforming Eq. (15) through the use of

$$\begin{Bmatrix} \bar{x}_N^B \\ x_I^B \\ \bar{x}_N^P \end{Bmatrix} = \begin{bmatrix} \phi_N^B & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & \bar{\phi}_N^P \end{bmatrix} \begin{Bmatrix} \bar{q}_N^B \\ x_I \\ \bar{q}_N^P \end{Bmatrix} \tag{16}$$

However, interface normal modes can also be defined by using the interface diagonal terms of Eq. (15), giving the transformation

$$\begin{Bmatrix} \bar{x}_N^B \\ x_I^B \\ \bar{x}_N^P \end{Bmatrix} = \begin{bmatrix} \bar{\phi}_N^B & 0 & 0 \\ 0 & \phi_I^B & 0 \\ 0 & 0 & \bar{\phi}_N^P \end{bmatrix} \begin{Bmatrix} \bar{q}_N^B \\ q_I \\ \bar{q}_N^P \end{Bmatrix} \tag{17}$$

When the transformation of Eq. (17) is applied to Eq. (15), the resulting modal system equations of motion may be written as

$$\begin{bmatrix} I & \bar{\phi}_N^{B^T} I_B^T M_B T_B \phi_I^B & 0 \\ \phi_I^{B^T} T_B^T M_B I_B \bar{\phi}_N^B & I & \phi_I^{B^T} T_P^T M_P I_P \bar{\phi}_N^P \\ 0 & \bar{\phi}_N^{P^T} I_P^T M_P T_P \phi_I^B & I \end{bmatrix} \begin{Bmatrix} \ddot{\bar{q}}_N^B \\ \ddot{q}_I^B \\ \ddot{\bar{q}}_N^P \end{Bmatrix} + \begin{bmatrix} [\bar{\omega}_B^2] & 0 & 0 \\ 0 & [\omega_I^2] & 0 \\ 0 & 0 & [\bar{\omega}_P^2] \end{bmatrix} \begin{Bmatrix} \bar{q}_N^B \\ q_I^B \\ \bar{q}_N^P \end{Bmatrix} = \begin{Bmatrix} \bar{\phi}_N^{B^T} I_B^T F_B \\ \phi_I^{B^T} T_B^T F_B \\ 0 \end{Bmatrix} \quad (18)$$

$[\bar{\omega}_B^2]$, $[\omega_I^2]$, and $[\bar{\omega}_P^2]$ are the eigenvalues based, respectively, on the three diagonal partitions of the mass and stiffness matrices of Eq. (15).

Component mode synthesis can also be used to solve Eq. (12) by solving a booster eigenproblem to determine booster free-interface normal modes, $[\phi_B]$, given by

$$[\phi_B] = \begin{bmatrix} \phi_N^B \\ \phi_I^B \end{bmatrix} \quad (19)$$

and again employing fixed-interface normal modes, $[\bar{\phi}_N^P]$, for the payload. The coordinate transformation becomes

$$\begin{Bmatrix} x_N^B \\ x_I^B \\ \bar{x}_N^P \end{Bmatrix} = \begin{bmatrix} \phi_N^B & 0 \\ \phi_I^B & 0 \\ 0 & \bar{\phi}_N^P \end{bmatrix} \begin{Bmatrix} q_B \\ \bar{q}_N^P \end{Bmatrix} \quad (20)$$

and the resulting modal equations of motion for the free booster-constrained payload model are given by

$$\begin{bmatrix} I + \phi_I^{B^T} T_P^T M_P T_P \phi_I^B & \phi_I^{B^T} T_P^T M_P I_P \bar{\phi}_N^P \\ \bar{\phi}_N^{P^T} I_P^T M_P T_P \phi_I^B & I \end{bmatrix} \begin{Bmatrix} \ddot{q}_B \\ \ddot{\bar{q}}_N^P \end{Bmatrix} + \begin{bmatrix} [\omega_B^2] + \phi_I^{B^T} T_P^T K_P T_P \phi_I^B & 0 \\ 0 & [\bar{\omega}_P^2] \end{bmatrix} \begin{Bmatrix} q_B \\ \bar{q}_N \end{Bmatrix} = \begin{Bmatrix} \phi_N^{B^T} F_N^B \\ 0 \end{Bmatrix} \quad (21)$$

where $[\omega_B^2]$ contains the eigenvalues of the booster with free interface and, as before, $[\omega_P^2]$ contains the eigenvalues of the payload with constrained interface.

Hou[18] proposed a component mode synthesis procedure in which both booster and payload would have free interfaces, and MacNeal[19] proposed a hybrid method which admits the free booster-constrained payload model here as a special case.

Equations (18) and (21), as they are, do not yield any advantage over a direct solution of Eqs. (15) and (12). However, in most practical applications there is a possibility of defining a so-called cut-off frequency. In these cases a Fourier series expansion of the force vector $\{F_N^B\}$ shows that the energy content of the high-frequency components is small compared to the energy contained in the low-frequency components. Practically, this means that the response of the structure due to the high-frequency content of $\{F_N^B\}$ can often be neglected. In this connection it should be noted that it is relatively difficult to excite the higher modes of the structure to any large extent, especially when $\{F_N^B\}$ only contains a few elements (i.e., only a few application points). The idea then is to retain only those modes in Eqs. (18) and (21) that have a frequency smaller than the cut-off frequency. This, in turn, reduces the size of Eqs. (18) and (21) considerably. Experience has shown that the introduction of a cut-off frequency is a workable concept.

At this point the standard practice is to solve for the so-called modal modes and frequencies, i.e., one solves the *system* eigenvalue problem corresponding to Eq. (18) or (21) after the booster and payload modes have been truncated according to the cut-off frequency.

This leads to a set of uncoupled modal system equations. In most cases, the system modes again are truncated according to the cut-off frequency. Then, a numerical scheme (e.g., Runge-Kutta) is used to obtain the modal response of the system. This response is then used to calculate loads from equations which are derived from Eq. (1).

The above approach works very well when applied to Eq. (18). However, some difficulties arise when Eq. (21) is used, i.e., when a free booster is used. Truncation of the high-frequency modes may have a significant effect on the *static* flexibility of the interface where the payload connection forces are applied. In the case of Eq. (18), this problem does not arise if *all* of the interface modes are retained in $[\phi_I^B]$.

There are two established methods for improving the interface representation in the model for the booster to improve upon Eq. (21). These involve, respectively, the use of interface loading and residuals.

B. The Mass and Stiffness Loading Technique

The free-interface normal modes employed in Eqs. (19-21) are obtained by solving the eigenproblem corresponding to

$$[M_B]\{\ddot{x}_B\} + [K_B]\{x_B\} = \{0\} \quad (22)$$

In more expanded form this equation appears as the terms involving $[M_B]$ and $[K_B]$ in Eq. (12). The interface loading technique, which was developed by Benfield and Hruda,[20] involves including the $[T_P^T M_P T_P]$ and $[T_P^T K_P T_P]$ terms in Eq. (12) as added mass and stiffness terms associated with the interface coordinates when solving the booster eigenproblem.

The main difference lies in the fact that in solving this new eigenvalue problem, the booster interface is mass and stiffness loaded by $[T_P^T M_P T_P]$ and $[T_P^T K_P T_P]$, respectively; i.e., the booster interface is loaded with approximate dynamic effects of the payload. Therefore, the new modes and frequencies may be expected to include a good representation of the interface. This permits a reduction of the number of booster modes according to the predetermined cut-off frequency. Also, it allows for faster convergence of the system eigenvalue problem if an iterative technique is used. The disadvantage of this method is that the eigenvalue problem is now dependent on the payload. This means that for every change in the payload this eigenvalue problem must be solved again. This is a disadvantage in cases where the booster does not change, e.g., in future Space Shuttle applications. However, if the changes in P are small, the old booster modes may be used as a first estimate to calculate the new booster modes in a Raleigh-Ritz-type eigenvalue problem solver.

C. Residual Stiffness and Mass Methods

The free-booster modes in Eqs. (19-21) may be separated into a set $[\phi_B^L]$ having frequencies less than, or equal to, the specified cut-off frequency, and a set $[\phi_B^H]$ of higher frequency modes, i.e.,

$$\{x_B\} = [\phi_B^L \mid \phi_B^H] \begin{Bmatrix} q_B^L \\ q_B^H \end{Bmatrix} \quad (23)$$

As noted before, simply truncating the modes by retaining only $[\phi_B^L]$ leads to poor accuracy in the response and the internal loads. MacNeal[19] presented a method for improving this representation using static approximations to the higher modes. This approach modifies the stiffness matrix by including residual flexibility effects. Rubin[21] derived a method for making further improvements through the addition of corrections to the mass matrix. Craig and Chang[8,15] gave a Ritz-type derivation of component mode synthesis methods which employ "attachment modes," including MacNeal's and Rubin's methods, and gave numerical comparisons of several free-interface component mode synthesis methods. A comprehensive discussion of these methods is also found in Ref. 13.

The principal advantage of these methods over the interface loading method previously described is that the computation of the residuals does not require any knowledge of the payload, so the booster analysis need not be repeated unless there are changes in the booster.

In order to retain the static contribution of the higher frequency modes, a set of residual attachment modes is introduced. These modes are employed to permit a more accurate flexibility representation at interface points and at other booster load points. The static (strictly, pseudostatic) booster response is given by

$$[K_B]\{x_B\} = \begin{Bmatrix} F_N^B \\ R_I^B \end{Bmatrix} \tag{24}$$

Substitution of Eq. (23) into Eq. (24) and premultiplication by $[\phi_B^T]$ gives

$$\left[\begin{array}{c|c} [\omega_B^{L^2}] & 0 \\ \hline 0 & [\omega_B^{H^2}] \end{array}\right] \begin{Bmatrix} q_B^L \\ q_B^H \end{Bmatrix} = \left[\begin{array}{c} \phi_B^{L^T} \\ \hline \phi_B^{H^T} \end{array}\right] \begin{Bmatrix} F_N^B \\ R_I^B \end{Bmatrix} \tag{25}$$

Thus, the pseudostatic contribution of the higher modes is given by the lower partition, namely,

$$\{q_B^H\} = [\omega_B^{H^{-2}}][\phi_B^H]^T \begin{Bmatrix} F_N^B \\ R_I^B \end{Bmatrix} \tag{26}$$

This can now be substituted into Eq. (23) to give

$$\{x_B\} = \left[\phi_B^L \;\middle|\; [\phi_B^H][\omega_B^{H^{-2}}][\phi_B^{H^T}]\right] \begin{Bmatrix} q_B^L \\ \hline F_N^B \\ R_I^B \end{Bmatrix} \tag{27}$$

The right-hand partition of the transformation matrix above constitutes the set of residual attachment modes, $[\phi_B^R]$.

$$[\phi_B^R] = [\phi_B^H][\omega_B^{H^{-2}}][\phi_B^{H^T}] \tag{28}$$

Although expressed here in terms of the higher modes and higher frequencies, residual attachment modes may be computed using only the static flexibility matrix and the lower modes and frequencies.[15,19]

The significant difference between Eqs. (23) and (27) is that the number of residual mode coordinates in Eq. (27) is only equal to the number of interface connection coordinates and the number of booster load points.

Further details of residual mass and stiffness methods may be found in Refs. 8 and 13.

D. The Coupled Base Motion Technique

The coupled base motion technique as developed by Dever et al.[22] is based on the separation of the booster response in Eq. (15) into two parts,

$$\begin{Bmatrix} \bar{x}_N^B \\ x_I^B \end{Bmatrix} = \begin{Bmatrix} \bar{x}_N^B \\ x_I^B \end{Bmatrix}^F + \begin{Bmatrix} \bar{x}_N^B \\ x_I^B \end{Bmatrix}^R \tag{29}$$

where the F part is due to the action of $\{F_B\}$ only and the R part is that due to the presence of the payload. In fact, the R part is the response due to payload feedback. It is clear that the F part satisfies Eq. (30),

$$\begin{bmatrix} I_B^T M_B I_B & I_B^T M_B T_B \\ T_B^T M_B I_B & T_B^T M_B T_B \end{bmatrix} \begin{Bmatrix} \ddot{\bar{x}}_N^B \\ \ddot{x}_I^B \end{Bmatrix}^F + \begin{bmatrix} I_B^T K_B I_B & 0 \\ 0 & T_B^T K_B T_B \end{bmatrix} \begin{Bmatrix} \bar{x}_N^B \\ x_I^B \end{Bmatrix}^F = \begin{Bmatrix} I_B^T F_B \\ T_B^T F_B \end{Bmatrix} \tag{30}$$

Note that the solution of Eq. (30) is a one-time computation effort as long as the booster and booster forces do not change. The R part satisfies Eq. (31).

$$\left[\begin{array}{c|c|c} I & \bar{\phi}_N^{B^T} I_B^T M_B T_B \phi_I^B & 0 \\ \hline \phi_I^{B^T} T_B^T M_B I_B \bar{\phi}_N^B & I & \phi_I^{B^T} T_P^T M_P I_P \bar{\phi}_N^P \\ \hline 0 & \bar{\phi}_N^{P^T} I_P^T M_P T_P \phi_I^B & I \end{array}\right] \begin{Bmatrix} \ddot{\bar{q}}_N^{BR} \\ \hline \ddot{q}_I^{BR} \\ \hline \ddot{\bar{q}}_N^P \end{Bmatrix} + \left[\begin{array}{c|c|c} [\bar{\omega}_B^2] & 0 & 0 \\ \hline 0 & [\omega_I^2] & 0 \\ \hline 0 & 0 & [\bar{\omega}_P^2] \end{array}\right] \begin{Bmatrix} \bar{q}_N^{BR} \\ \hline q_I^{BR} \\ \hline \bar{q}_N^P \end{Bmatrix}$$

$$= -\begin{Bmatrix} 0 \\ \hline \phi_I^{B^T} T_P^T M_P T_P \ddot{x}_I^{BF} \\ \hline \bar{\phi}_N^{P^T} I_P^T M_P T_P \ddot{x}_I^{BF} \end{Bmatrix} - \begin{Bmatrix} 0 \\ \hline \phi_I^{B^T} T_P^T K_P T_P x_I^{BF} \\ \hline 0 \end{Bmatrix} \tag{31}$$

where Eqs. (29) and (30) have been used, together with the modal transformations. At this point the solution could proceed as noted in Sec. IV.A. However, when feedback is small, Eq. (31) may be used to derive a base drive method, which will be described subsequently.

E. A Direct Integration Technique

The unique form of Eq. (18) permits a very efficient numerical integration procedure to be employed, thereby avoiding the expensive solution of a system eigenvalue problem.[23] Because no system eigenvalue problem is solved, one does not know a priori the highest system frequency in Eq. (18). Although schemes to determine the highest frequency in the system exist, this highest frequency may be erroneous and/or unnecessarily high because of modal truncation of the B and P models. Therefore, integration schemes such as Runge-Kutta are not applicable because these schemes require a step size consistent with the highest system frequency. Consequently, an adaptation of the Newmark-Chan-Beta integration scheme is used. This numerical technique uses a step size consistent with the highest frequency of interest (i.e., the cut-off frequency) while remaining stable. The efficiency of the technique is primarily dependent on the size of

$$\left[\bar{\phi}_N^{B^T} I_B^T M_B T_B \phi_I^B\right] \quad \text{and} \quad \left[\bar{\phi}_N^{P^T} I_P^T M_P T_P \phi_I^B\right]$$

both of which have the column size equal to the number of interface degrees of freedom. Therefore, this method works best for a relatively small number of interface degrees of freedom, which is the case in Shuttle payload applications. Some of the savings are also passed on to the loads calculations, so that the overall cost savings may be anticipated to be very high as compared to other full-scale methods. Details of the method may be found in Ref. 23.

V. Solution of the Equations of Motion by Frequency Domain Techniques

A solution for Eqs. (4-6) can also be sought in the frequency domain. In this case it is necessary to incorporate damping into Eq. (6) or to incorporate it as modal damping. Geering[24] presented a straightforward direct stiffness assembly of the dynamic stiffness matrices of two substructures noting that the only degrees of freedom required are the interface degrees of freedom and those subjected to external excitation. Geering considered the primary usefulness of his frequency domain coupling procedure to be in analyzing steady-state response to periodic loads or spectral response to random loads. Klosterman and McClelland[25,26] developed a similar substructure coupling technique, but included a transient response capability and emphasized the use of experimental frequency response data in formulating the substructure matrices.

The impedance technique developed by Payne[27] differs from the above frequency domain techniques in that it employs accelerations rather than displacements. It is basically a full-scale method in the sense that it does not make any assumptions concerning the size of the payload or the extent of the changes made in the payload. However, the method does avoid a full-scale solution of the coupled booster/payload equations of motion and is particularly suited to deal with small changes in the payload. The impedance technique is essentially a base drive method to be discussed later. It differs from that approach in the manner in which the interface accelerations $\{\ddot{x}_I^B\}$ are computed. Indeed, the interface accelerations are computed in the frequency domain instead of the discrete time domain thereby essentially converting a set of differential equations into a set of algebraic equations.

The approximation involved in the impedance technique is imbedded in the transformations to and from the frequency domain. If these transformations were exact, the method of determining $\{\ddot{x}_I(t)\}$ would be exact. The use of the fast Fourier transform (FFT) in obtaining the spectral data to be used in this technique presents some problems, and the problem of damping seems to magnify when working in the frequency domain. Reference 27 includes applications to a realistic booster/payload system demonstrating the nature of these FFT and damping problems. In general, however, enough correlation with the exact time-domain solution is apparent to warrant further investigation into possible improvements. Also, the impedance technique is extremely well suited for application to cases of multiple design changes. Indeed, the new interface accelerations can be derived directly from the old interface accelerations.

Since the frequency domain methods mentioned above do not require solution of an eigenproblem for the coupled set of modal equations, and since only excitation and interface freedoms are employed, these methods could have been classified as short-cut methods and included in the following section.

VI. Short-Cut Methods

In Sec. IV several modal coupling techniques were reviewed. All of these techniques necessitate the solution of coupled booster/payload equations without any approximations except modal truncation. This solution can be quite expensive, especially if it must be repeated several times, e.g., during a design effort. Although mass and stiffness changes during a design effort are often small, current practices used to design payload structures require a new full-scale solution every time such small changes in the payload are made. Thus, the need exists for short-cut methods. The term short-cut method implies that the method should be able to evaluate the effect of small changes to the payload in a relatively short time. First, a short-cut method should avoid the direct solution of the booster/payload system. Second, it should avoid, as much as possible, the interfacing between different organizations.

Several short-cut methods are described below. They may be classified as 1) base drive, 2) structural modification, 3) perturbation, and 4) shock spectrum techniques.

A. Base Drive Techniques

The bottom row partition of Eq. (15) forms the basis of one approach to base drive analysis. This may be written

$$I_P^T M_P I_P \ddot{\bar{x}}_N^P + I_P^T K_P I_P \bar{x}_N^P = -I_P^T M_P T_P \ddot{x}_I^B \qquad (32)$$

so that the relative motion of the payload may be determined from a knowledge of the interface accelerations. In applications of this equation it has been assumed that the interface accelerations of one payload/booster system can be employed to drive the base of a modified payload. Studies are currently underway to compare measured payload acceleration responses with those calculated using a base drive approach equivalent to Eq. (32).

The coupled base motion formulation of Eq. (31) can also serve as the basis of a base drive technique. From the bottom row partition of Eq. (31) is obtained the equation

$$\{\ddot{\bar{q}}_N^P\} + [\bar{\omega}_P^2]\{\bar{q}_N^P\} = -\{\bar{\phi}_N^{P^T} I_P^T M_P T_P \ddot{x}_I^{BF}\} - \{\bar{\phi}_N^{P^T} I_P^T M_P T_P \phi_I^B\}\{\ddot{q}_I^{BR}\} \qquad (33)$$

The second term on the right-hand side is the one that couples the payload equation to the rest of the system. The idea of a base drive short-cut method is to approximate $\{\ddot{q}_I^{BR}\}$ in such a way as to avoid the solution of the complete Eq. (31).

A fairly well established practice is to assume that the presence of the payload has no effect on the response of the booster, i.e., $\{\ddot{q}_I^{BR}\}=0$, or the feedback of the payload on the booster is negligible. This effectively uncouples the payload from the booster. The conditions under which such an approximation is valid remains an unanswered question, but when the payload mass is small compared to the total mass of the system, experience shows this often to be a valid assumption. Equation (33) becomes

$$\{\ddot{\bar{q}}_N^P\} + [\bar{\omega}_P^2]\{\bar{q}_N^P\} = -\{\bar{\phi}_N^{P^T} I_P^T M_P T_P \ddot{x}_I^{BF}\} \qquad (34)$$

which means that the payload is directly driven at its base by the known excitation on the right-hand side of Eq. (34).

Chen et al.[28] proposed a recovered transient load analysis method which is also a base drive technique. The method employs a transient analysis with a reference payload, recovers from it the interface accelerations for the unloaded launch vehicle, and then modifies this interface motion to include the dynamic effects of a new payload. The procedure does not involve the detailed launch vehicle model or launch vehicle forcing functions; hence the entire process can be implemented by the payload organization. The development in Ref. 28 is limited to a statically determinate interface.

B. Structural Modification Techniques

Recently, several software products have become available for analyzing the effect of structural modification on structural response, usually in the frequency domain.[25,29] These, however, are more applicable as "troubleshooting" tools than as design tools, since they employ experimental data. Of more interest for payload design are methods which are efficient for analyzing the effect of payload design changes. Such changes include 1) alteration of the mass and stiffness in the "change region" leaving the topology unchanged, 2) removal of the change region structure completely, and 3) removal of the change region structure and replacement with a completely new structure.

Coale and White[30] developed a structural modification technique that is capable of assessing the effect of the above types of design changes efficiently. The modes of the modified system are expressed in terms of the old modes augmented by a set of constraint modes associated with the modified area. The new eigenvalue problem displays the mass and stiffness matrix of the modified area explicitly so that it can be changed arbitrarily. No complete new solution is necessary, thereby reducing the cost of the analysis.

A similar approach to structural modification was developed by Coppolino,[31] who applied the technique to conduct vibration mode reanalysis of a locally damaged or altered structure. The method employs a truncated set of original structure modes, augmented by an appropriate set of residual attachment vectors, to form a reduced description of the damaged structure. The desired lower frequency modes of the locally changed structure are computed using the reduced equations. One application to offshore structures cited by the author produced natural frequencies which were accurate to within 0.2% at a computational cost of approximately 4% of the cost of a complete reanalysis.

Although the above structural modification methods hold great promise as efficient design reanalysis tools, they have not yet been thoroughly tested and compared.

C. Perturbation Techniques

Matrix perturbation techniques have been the subject of extensive development at the Jet Propulsion Laboratory.[32,33] In contrast to the structural modification techniques discussed above, the matrix perturbation approach assumes that the structural modifications are "small," i.e.,

$$[M]=[M_0]+\epsilon[M_1], \qquad [K]=[K_0]+\epsilon[K_1] \quad (35)$$

The eigenvectors and eigenvalues are approximated by

$$[\phi]=[\phi_0]+\epsilon[\phi_1], \qquad [\omega]=[\omega_0]+\epsilon[\omega_1] \quad (36)$$

If the reference solution is orthonormalized so that

$$[\phi_0]^T[M_0][\phi_0]=[I], \qquad [\phi_0^T][K_0][\phi_0]=[\omega_0^2] \quad (37)$$

then the perturbation technique of Ref. 33 may be summarized by the following equations. Introducing power series in ϵ for all relevant quantities leads to a set of equations

$$\{\ddot{q}_0\}+[\omega_0^2]\{q_0\}=[\phi_0]^T\{F\} \quad (38)$$

$$\{\ddot{q}_1\}+[\omega_0^2]\{q_1\}=[\phi_1]^T\{F\}-2[\omega_0][\omega_1]\{q_0\} \quad (39)$$

which can be solved in sequence, once $[\omega_1]$ and $[\phi_1]$ are known.

It is always possible to write $[\phi_1]$ as a linear combination of the complete set of eigenvectors $[\phi_0]$,

$$[\phi_1]=[\phi_0][\alpha] \quad (40)$$

Then, the expansion coefficient matrix $[\alpha]$ and the eigenvalue perturbation matrix $[\omega_1]$ can be obtained by solving the equations simultaneously

$$[\alpha]+[\alpha]^T=-[\phi_0]^T[M_1][\phi_0] \quad (41)$$

and

$$2[\omega_0][\omega_1]=[\omega_0^2][\alpha]+[\alpha]^T[\omega_0^2]+[\phi_0]^T[K_1][\phi_0] \quad (42)$$

Several questions regarding the limits of validity of the above method may be raised: 1) Must *each* element in the modified stiffness and mass matrices differ from the reference values by a term of "order ϵ?" 2) What changes are required if the original system has identical, or very closely spaced, frequencies? 3) Are dynamic responses of the perturbed structure computed as accurately as the eigenvalues and eigenvectors?

Higher-order perturbations have been considered by Ryland and Meirovitch,[34] and Caughey[32] has considered systems having repeated eigenvalues. To the authors' knowledge, perturbation methods have not been compared with the more general modification methods such as Refs. 30 and 31.

D. A Generalized Shock Spectrum Technique

Whereas the previous two subsections have discussed techniques for determining the natural frequencies and mode shapes of modified structures, the present section describes a technique for estimating maximum response quantities directly.[35] Equation (21) serves as the basis of this technique. The basic idea of the generalized shock spectrum technique is to determine the load maxima without having to solve Eq. (21) explicitly. To reach this goal, a new model is introduced for both the coupled system and the forcing function. Assume that M free-booster modes are retained in $[\phi_B]$ and N modes are retained for the cantilevered payload.

First, the $(N+M)$ coupled equations in Eq. (21) are replaced by $(N\times M)$ sets of two simultaneous equations each of which represents the coupling of one payload mode with one booster mode, as follows,

$$\begin{bmatrix} 1+\{\phi_I^B\}_i^T T_P^T M_P T_P\{\phi_I^B\}_i & \{\phi_I^B\}_i^T T_P^T M_P I_P\{\bar{\phi}_N^P\}_j \\ \{\bar{\phi}_N^P\}_j^T I_P^T M_P T_P\{\phi_I^B\}_i & 1 \end{bmatrix} \begin{Bmatrix} \ddot{q}_{Bi} \\ \ddot{\bar{q}}_{Nj} \end{Bmatrix} + \begin{bmatrix} \omega_{Bi}^2 & 0 \\ 0 & \bar{\omega}_{Pj}^2 \end{bmatrix} \begin{Bmatrix} q_{Bi} \\ \bar{q}_{Nj}^P \end{Bmatrix} = \begin{Bmatrix} \{\phi_N^B\}_i^T \; F_N^B \\ 0 \end{Bmatrix} \quad \begin{matrix} i=1,2,\ldots,M \\ j=1,2,\ldots,N \end{matrix} \quad (43)$$

In the preceding equation it has been assumed that the interface is statically determinate, i.e., $[T_P^T K_P T_P]=[0]$.

Second, a bound q_{BP} on each of the $(N\times M)$ modal responses of the payload is established. This is done by introducing a new model for the forcing function in Eq. (43). The rather complicated forcing function is replaced by a much simpler function (e.g., an impulsive force) which produces the same maximum response peak as the original force. Therefore, an analytical solution for both the response and maximum response of Eq. (43) is possible (after some additional simplifications). Finally, a bound q_P on the total payload response is constructed by summing over all the individual modal bounds q_{BP} (over absolute values or in a root-sum-square sense that allows for phase weighting). Payload member loads are obtained by adding the contributions of all payload modal loads.

As stated above, the forcing function in Eq. (43) is replaced by the modal delta function of a certain magnitude F_B. This magnitude F_B is evaluated from an already existing transient analysis of the booster with or without a dummy payload. The shock spectrum technique shows some very attractive features such as low cost and fast turnaround time. However, the method leads to rather conservative values for the loads (at least in the case of the Voyager). Obviously, this is due to the replacement of the actual structural model and force model by new approximate models. No proof is given that such changes always lead to conservatism and no under design is possible. The shock spectrum method essentially represents a tradeoff between cost and time on the one hand and increase in structural weight on the other. This is a point that must be considered for every payload. The authors feel that the shock spectrum approach can be valuable in cases where structural weight is not a primary concern (e.g., earthquake analysis) or in obtaining approximate preliminary design loads in the early stages of payload development.

VII. Conclusions

Several full-scale and short-cut methods to analyze a booster/payload system have been presented. In the category of full-scale methods two approaches are currently being used extensively. The first technique uses a restrained payload together with a free-booster model, the latter being augmented with the residual mass and stiffness correction. The second technique uses a restrained payload and booster model. Both techniques determine the so-called "modal modes," which require the solution of a system eigenvalue problem. The loads usually are then determined employing an acceleration approach. These techniques are well established and yield accurate results. They are, however, rather expensive. A direct numerical integration approach recently introduced can lead to significant computational savings if the number of interface degrees of freedom is small.

A number of "short-cut" methods also have been described briefly. Of special interest to Shuttle payload design are base drive, structural modification, and interface impedance methods. Because of the need to reduce computational cost and time while maintaining computational accuracy, further research is needed in developing and comparing these short-cut methods. Specifically, base drive methods should be evaluated and compared noting, in particular, what flight data should be acquired to form a database for the use of base drive in payload design. The structural modification methods should be compared, and computational costs for complete payload reanalysis determined. Finally, frequency domain methods should be developed further and compared with time-domain approaches.

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